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MATDIP301

Third Semester B.E. Degree Examination, July/August 2022
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the modulus and amplitude of $\dfrac{1}{1-\cos\alpha+i\sin\alpha}$. **(07 Marks)**

b. Express $\dfrac{(3+i)(1-3i)}{2+i}$ in the form of x + iy. **(06 Marks)**

c. Find the cube roots of 1-i **(07 Marks)**

2 a. Find the n^{th} derivative of $\dfrac{x}{(x+1)(2x+1)}$. **(07 Marks)**

b. Find the n^{th} derivative of cosx cos3x cos5x. **(06 Marks)**

c. If $y = a\cos(\log x) + b\sin(\log x)$ show that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **(07 Marks)**

3 a. Show that the following pairs of curves intersect orthogonally $r_1 = a(1+\sin\theta)$ and $r_2 = a(1-\sin\theta)$. **(07 Marks)**

b. With usual notations prove that $\tan\phi = r\dfrac{d\theta}{dr}$. **(06 Marks)**

c. Expand $\sqrt{1+\sin 2x}$ by Maclaurin's series upto the term containing x^4. **(07 Marks)**

4 a. State and prove Euler's theorem on homogeneous functions. **(07 Marks)**

b. If $u = f(x-y, y-z, z-x)$ show that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$. **(06 Marks)**

c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$ find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1, -1, 0). **(07 Marks)**

5 a. Obtain a reduction formula for $I_n = \int_0^{\pi/2}\cos^n x\,dx$, n being a positive integer. **(07 Marks)**

b. Evaluate $\int_0^1\int_x^{\sqrt{x}}(x^2+y^2)\,dy\,dx$. **(06 Marks)**

c. Evaluate $\int_{-1}^{1}\int_0^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$. **(07 Marks)**

6 a. Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$. **(07 Marks)**

b. Express $I=\int_0^1 x^{3/2}(1-x)^{1/2}dx$ in terms of beta function. **(06 Marks)**

c. Show that $\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}}\times\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta=\pi$. **(07 Marks)**

7 a. Solve: $(2x+y+1)dx+(x+2y+1)dy=0$. **(07 Marks)**

b. Solve $\frac{dy}{dx}+y\cot x=\cos x$. **(06 Marks)**

c. Solve: $\sec^2x\tan y\,dx+\sec^2y\tan x\,dy=0$. **(07 Marks)**

8 a. Solve: $6\frac{d^2y}{dx^2}+17\frac{dy}{dx}+12y=e^{-x}$. **(07 Marks)**

b. Solve: $4\frac{d^4y}{dx^4}-8\frac{d^3y}{dx^3}-7\frac{d^2y}{dx^2}+11\frac{dy}{dx}+6y=0$. **(06 Marks)**

c. Solve: $\frac{d^3y}{dx^3}-y=3\cos 2x$. **(07 Marks)**

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